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by Ozanam,[1] but it is in fact much older, having been announced by Nicolaus Cusanus.[2] It has been discussed by various writers,[3] with regard to its proof and estimates of its accuracy; the references were not accessible to Professor Johnson. Although his proof is essentially the same as that in Mansion's first paper, cited in the footnote, it has seemed worth while to publish it, for the sake of calling attention to this remarkable approximation, which does not seem to be familiar to most mathematicians.

By equating to zero the derivative of the expression

$$\frac{172\sin B}{2+\cos B}-\frac{180B}{\pi}$$

we find that the maximum positive deviation occurs for $B = 22^\circ\ 20'$ approximately, and is equal to about 0.0122° or less than 45″. The maximum negative deviation, as indicated by the author's tables, is 0.0730° or between 4′ and 5′.

I. Note on the Solution of Fractional Equations.

By Charles A. Noble, University of California.

In Wilczynski and Slaught's *College Algebra* the following interesting statement occurs on page 242:

"If, however, we express every one of the rational fractions as a sum of simple partial fractions, then unite all of the partial fractions which have the same denominator into a single one, and finally add these partial fractions together, using as a common denominator the lowest common multiple of the denominators of the simple partial fractions, we may be sure that the sum obtained in this way is in its lowest terms."

The correctness of this rule was considered evident by the authors and the proof of it was omitted. The following is a formal proof:

Let F, G, P, and Q be polynomials in x. Assume first that G and Q have only one common linear factor, and that each has only one other linear factor, where each factor may be present in any order of multiplicity. Let F/G and P/Q, assumed to be in their lowest terms, be expressed in partial fractions:

$$\frac{F}{G}=\frac{A_1}{(x-\alpha)^k}+\frac{A_2}{(x-\alpha)^{k-1}}+\cdots+\frac{A_k}{x-\alpha}+\frac{B_1}{(x-\beta)^\lambda}+\frac{B_2}{(x-\beta)^{\lambda-1}}+\cdots+\frac{B_\lambda}{x-\beta},$$

$$\frac{P}{Q}=\frac{C_1}{(x-\beta)^\mu}+\frac{C_2}{(x-\beta)^{\mu-1}}+\cdots+\frac{C_\mu}{x-\beta}+\frac{D_1}{(x-\gamma)^\nu}+\frac{D_2}{(x-\gamma)^{\nu-1}}+\cdots+\frac{D_\nu}{x-\gamma}.$$

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[1] *Nouvelle trigonométrie, où l'on trouve le moyen de calculer toutes sortes de triangles rectilignes sans les tables de sinus, . . .,* 1699.

[2] *De mathematica perfectione; Opera,* Paris, 1514.

[3] F. A. Protche, *Mémoires de la société académique de l'Aube,* Vol. 51 (1887), pp. 149–162. H. Brocard, *Mathesis,* 1889, p. 161; P. Mansion, ibid., p. 162; P. Mansion, ibid., p. 181. Further references on closely related formulas may be found in Th. Vahlen, *Geometrische Konstruktionen und Approximationen,* Leipsic, 1911, pp. 188–206.

Assume $\lambda \geqq \mu$ and add:

$$\frac{F}{G}+\frac{P}{Q}$$

$$= \frac{(x-\beta)^{\lambda}(x-\gamma)^{\nu}\{A_1+A_2(x-\alpha)+\cdots\}+(x-\alpha)^{k}(x-\gamma)^{\nu}\{B_1+B_2(x-\beta)+\cdots\}}{(x-\alpha)^{k}(x-\beta)^{\lambda}(x-\gamma)^{\nu}}$$

$$+\frac{(x-\alpha)^{k}(x-\beta)^{\lambda-\mu}(x-\gamma)^{\nu}\{C_1+C_2(x-\beta)+\cdots\} + (x-\alpha)^{k}(x-\beta)^{\lambda}\{D_1+D_2(x-\gamma)+\cdots\}}{(x-\alpha)^{k}(x-\beta)^{\lambda}(x-\gamma)^{\nu}}.$$

Now the sum on the right, with the common denominator $(x-\alpha)^k(x-\beta)^\lambda(x-\gamma)^\nu$ will be in its lowest terms unless its numerator has $x - \alpha$, $x - \beta$, or $x - \gamma$ as a factor. $x - \alpha$ will be a factor if, and only if, $A_1 = 0$. But this implies that F/G is not in its lowest terms. Similarly, $x - \gamma$ will be a factor if, and only if, $D_1 = 0$, which is not possible if P/Q is in its lowest terms. If $\lambda > \mu$, $x - \beta$ will be a factor if, and only if $B_1 = 0$, which also contradicts the assumption that F/G is in its lowest terms. If $\lambda = \mu$, $x - \beta$ will be a factor if $B_1 = 0$, $C_1 = 0$, which is again contrary to assumption; but it is also a factor if $B_1 = - C_1$, and this is possible, even when F/G and P/Q are both in their lowest terms. In this case the rule given above would remove the terms in B_1 and C_1, and all succeeding pairs of terms such that $B_i + C_i = 0$, before the sum is reduced to a common denominator.

The proof for the general case, where the sum is of n rational fractions, instead of two, and where the denominators contain $m_1, m_2, \cdots m_n$ binomial factors instead of two, is an obvious extension of the above.

When a fractional equation

$$\frac{F}{G}+\frac{P}{Q}=0$$

is to be solved, it may not be apparent whether F/G and P/Q are in their lowest terms. Resolution into partial fractions will disclose the fact. If they are in their lowest terms, the usual method of solution, by reducing to lowest common denominator and equating to zero the sum of the numerators, will not yield an extraneous solution unless, in the above notation, $B_1 + C_1 = 0$; and the method suggested by the theorem just proved obviates this difficulty.

Example:

$$\frac{F}{G}=\frac{2x^2-4x+1}{(x-1)^2(x-2)}=\frac{1}{(x-1)^2}+\frac{1}{x-1}+\frac{1}{x-2},$$

$$\frac{P}{Q}=\frac{5x^2-15x+12}{(x-1)^2(x-3)}=\frac{-1}{(x-1)^2}+\frac{2}{x-1}+\frac{3}{x-3}.$$

Here both of the given fractions are in their lowest terms. The usual method of solving

$$\frac{F}{G}+\frac{P}{Q}=0$$

would be

$$\frac{F}{G}+\frac{P}{Q}=\frac{7x^3-35x^2+55x-27}{(x-1)^2(x-2)(x-3)}=0,$$

from which $7x^3-35x^2+55x-27=0$, or $x=1, 2\pm\frac{1}{7}\sqrt{7}$.

The root $x=1$ is extraneous. The sum of F/G and P/Q is not in its lowest terms. The reason is that $C_1=-B_1$. If the rule stated above is followed, one gets

$$\frac{F}{G}+\frac{P}{Q}=\frac{3}{x-1}+\frac{1}{x-2}+\frac{3}{x-3}=\frac{7x^2-28x+27}{(x-1)(x-2)(x-3)}=0,$$

which yields the true roots $x=2\pm\frac{1}{7}\sqrt{7}$.

II. Determination of an Angle of a Right Triangle, without Tables.

By Roger A. Johnson, Hamline University.

If a, b, c denote respectively the longer leg, the shorter leg, and the hypotenuse, then the value, in degrees, of the smaller acute angle is given approximately by

$$B=172\frac{b}{a+2c}.$$

This formula was given by Ozanam, 1699.

In practice it often happens that all three sides of a right triangle are known, and it is desired to find the angles quickly; the above formula will give them without the use of tables, with almost four-place accuracy for angles up to 35°, and better than three-place accuracy up to 45°. When the sides are given in integers, this method is simpler than the ordinary one. Again, it is useful as a check-formula in testing solutions.

The proof is easily effected by means of Taylor's Series. If we write

$$\frac{b}{a+2c}=\frac{\sin B}{2+\cos B}=f(B),$$

the expansion of $f(B)$ is easily found to be

$$f(B)=\frac{B}{3}\left(1-\frac{1}{180}B^4-\frac{1}{1512}B^6\cdots\right),$$

whence it is evident that for small values of B, $f(B)$ is nearly equal to $\frac{1}{3}B$ radians. That is, if we desire to have B in degrees,

$$B\text{ (degrees)}=\frac{3\times 180}{\pi}f(B)+e,$$

where e is a small correction. We may correct the error in part by replacing the coefficient $540/\pi$, or $171.89\cdots$, by the simpler number 172. The degree of